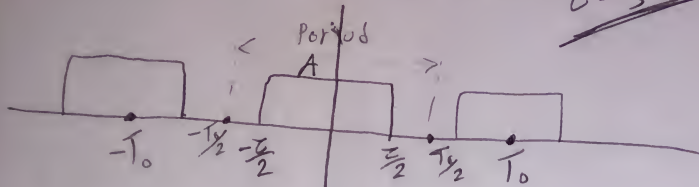


Sec 2

2 marks

\* Find and sketch the magnitude spectrum of the following functions using Complex F.S and calculate the avg. power  $T = \frac{1}{3} T_0$



$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p(t) e^{-jn\omega_0 t} dt$$

$$|C_n| = \sqrt{a_n^2 + b_n^2}$$

$$g_p(t) = \begin{cases} A & -\frac{T_0}{6} \leq t \leq \frac{T_0}{6} \\ 0 & \text{otherwise} \end{cases}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/6}^{T_0/6} A e^{-jn\omega_0 t} dt$$

$$= \frac{A}{T_0} \cdot \frac{1}{-jn\omega_0} \left[ e^{-jn\omega_0 t} \right]_{-T_0/6}^{T_0/6}$$

$$= \frac{A}{T_0} \cdot \frac{1}{-jn\omega_0} \left[ e^{-jn\omega_0 \frac{T_0}{6}} - e^{jn\omega_0 \frac{T_0}{6}} \right]$$

Note

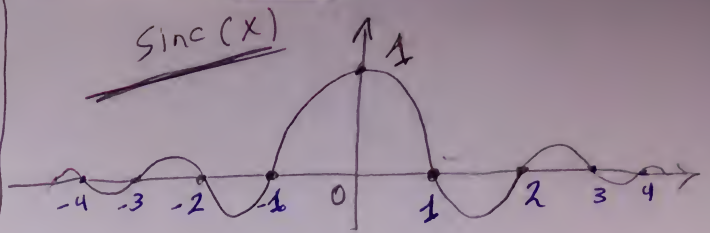
$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$= \frac{2A}{2\pi n} \left[ \frac{e^{jn\omega_0 \frac{T_0}{6}} - e^{-jn\omega_0 \frac{T_0}{6}}}{2j} \right]$$

$$C_n = \frac{A}{n\pi} \sin \left( n\omega_0 \frac{T_0}{6} \right)$$

Note

$$\text{Sinc}[x] = \frac{\sin(\pi x)}{\pi x}$$



$\text{Sinc}(x) = 0$  when  $x$  is integer and  $x \neq 0$

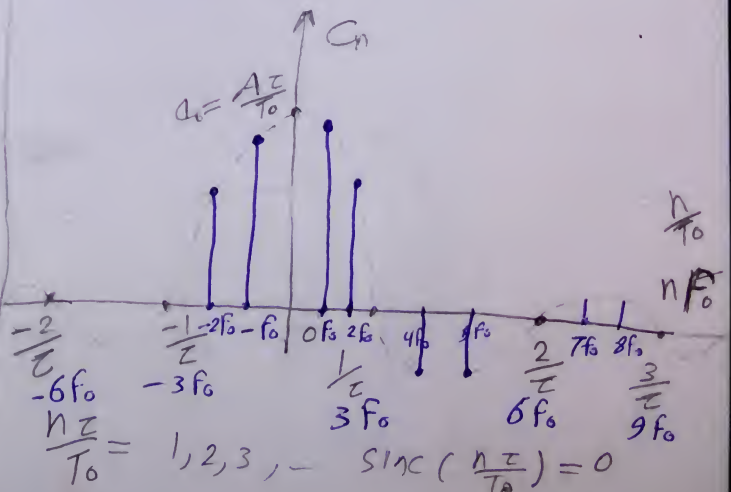
$$C_n = \frac{A}{n\pi} \sin \left( n \frac{2\pi}{T_0} \frac{T_0}{6} \right)$$

$$C_n = \frac{A T}{T_0} \times \frac{\sin \left( \frac{n\pi T}{T_0} \right)}{\frac{n\pi T}{T_0}}$$

$$C_n = \frac{A T}{T_0} \text{Sinc} \left( \frac{n T}{T_0} \right)$$

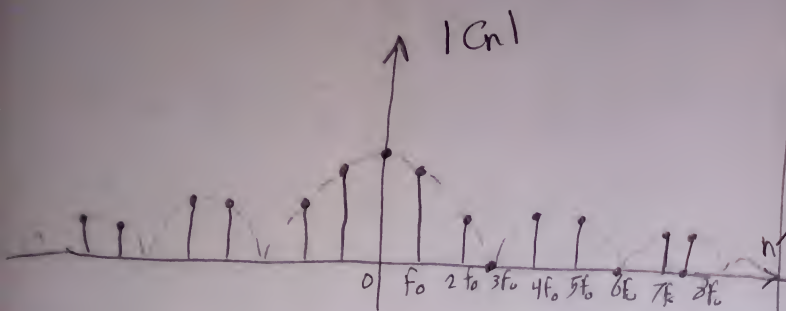
$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{+jn\omega_0 t}$$

$$= \frac{A T}{T_0} \sum_{n=-\infty}^{\infty} \text{Sinc} \left( \frac{n T}{T_0} \right) e^{+jn\omega_0 t}$$



$$\frac{n T}{T_0} = \frac{1}{2}, \frac{2}{2}$$

$$C_n = |C_n| \cdot e^{j\theta_n}$$

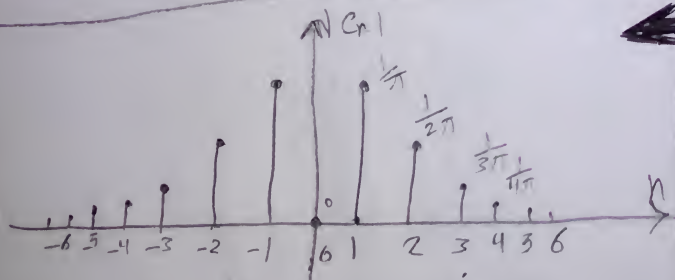


$$* P_{avg} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g^2(t) dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 dt$$

$$= \frac{A^2}{T_0} \cdot t \Big|_{-T_0/2}^{T_0/2}$$

$$P_{avg} = \frac{A^2 T}{T_0} \text{ watts}$$



$$g_p(t) = \sum_{n=-\infty}^{\infty} \frac{j}{n\pi} e^{jn\omega_0 t}$$

$$P_{avg} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{t^2}{\pi^2} dt$$

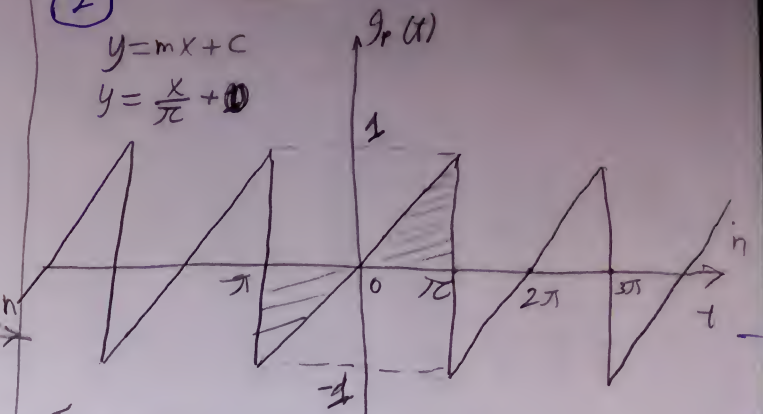
$$= \frac{1}{3 \times 2\pi^3} |t^3|_{-\pi}^{\pi}$$

$$= \frac{1}{3} \text{ watt}$$

(2)

$$y = mx + c$$

$$y = \frac{x}{\pi} + 0$$



$$① T_0 = 2\pi \text{ from } -\pi \rightarrow \pi$$

$$② g_p(t) = \frac{t}{\pi} \quad -\pi \leq t \leq \pi$$

$$③ g_p(t) \text{ is odd } a_n = 0, a_0 = 0$$

$$b_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p(t) \sin(n\omega_0 t) dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{t}{\pi} \sin(n\omega_0 t) dt$$

$$\omega_0 = \frac{2\pi}{T_0} = 1$$

$$= \frac{1}{2\pi^2} \left[ \frac{-t \cos(n\omega_0 t)}{n\omega_0} + \int \frac{\cos(n\omega_0 t)}{n\omega_0} dt \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi^2} \left[ \frac{-\pi \cos(n\omega_0 \pi)}{n\omega_0} - \frac{\pi \cos(n\omega_0 (-\pi))}{n\omega_0} + \frac{\sin(n\omega_0 t)}{n^2 \omega_0^2} \Big|_{-\pi}^{\pi} \right]$$

$$\sin(n\pi) = 0$$

$$\cos(n\pi) = (-1)^n$$

$$= \frac{1}{2\pi^2} \left[ \frac{-2\pi \cos(n\pi)}{n\omega_0} + 0 \right]$$

$$b_n = \frac{(-1)^{n+1}}{n\pi}$$

$$C_n = |C_n| e^{j\theta_n}$$

$$\theta_n = \tan^{-1} \frac{b_n}{a_n}$$

$$\theta_n = \frac{\pi}{2}$$

$$C_n = j \frac{1}{n\pi}$$

$$|C_n| = |b_n| = \frac{1}{n\pi}$$

$$|C_1| = \frac{1}{\pi}$$

$$|C_2| = \frac{1}{2\pi}$$

$$|C_3| = \frac{1}{3\pi}$$

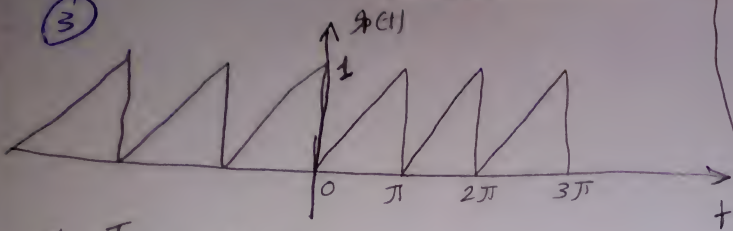


2

Sec 2

حل المسألة

3



$$* T_0 = \pi \text{ from } 0 \rightarrow \pi$$

$$* g_p(t) = \frac{t}{\pi}$$

\* not odd nor even

$$\omega_0 = \frac{2\pi}{T_0} = 2$$

$$C_n = \frac{1}{T_0} \int_0^{T_0} g_p(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{\pi^2} \int_0^{\pi} t e^{-jn\omega_0 t} dt$$

$$u = t \quad dv = e^{-jn\omega_0 t}$$

$$du = 1 \quad v = \frac{1}{-jn\omega_0} e^{-jn\omega_0 t}$$

$$C_n = \frac{1}{\pi^2} \left[ \frac{-t}{jn\omega_0} e^{-jn\omega_0 t} + \int_0^{\pi} \frac{e^{-jn\omega_0 t}}{jn\omega_0} dt \right]_0^{\pi}$$

$$C_n = \frac{1}{\pi^2} \left[ \left( \frac{-\pi}{jn\omega_0} e^{-j2n\pi} \right) + \frac{1}{jn\omega_0} \left( \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right) \right]_0^{\pi}$$

$$C_n = \frac{1}{\pi^2} \left[ \left( \frac{-\pi}{jn\omega_0} e^{-j2n\pi} \right) - \frac{1}{4n^2} (e^{-j2n\pi} - 1) \right]$$

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

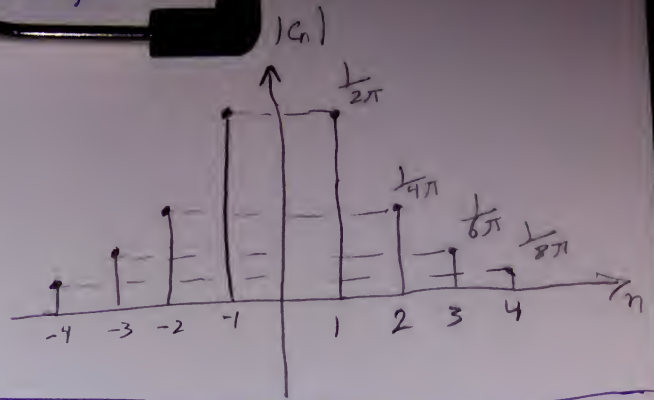
$$e^{-jn\pi} = \cos\left(\frac{2n\pi}{2}\right) - j \sin\left(\frac{2n\pi}{2}\right)$$

$\rightarrow 1$

$$C_n = \frac{1}{\pi^2} \left[ \frac{-\pi}{jn\omega_0} \right]$$

$$C_n = \frac{-1}{jn\omega_0} = \frac{j}{2n\pi}$$

$$|C_n| = \frac{1}{2n\pi}$$



$C_n$  عبارة عن 2 جزء دالة زوجية

$$P_{avg} = \frac{1}{\pi} \int_0^{\pi} \frac{t^2}{\pi^2} dt$$

$$= \frac{1}{3\pi^3} \left[ t^3 \right]_0^{\pi}$$

$$= \frac{1}{3}$$